

Eynesbury Church of England Primary School



Reasoning and Problem Solving Progression Document

Progression in Reasoning and Problem Solving at Eynesbury Church of England Primary School

The National Curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions¹

This progression map is written to help meet these aims, to promote mathematical reasoning in children, to develop an ability to convince others using mathematical arguments, and to engage with open ended problem solving.

Mathematical reasoning and problem solving are best embedded in all lessons; the learning outcomes and activities suggested are recommended as part of everyday teaching, though when teaching problem solving skills discrete lessons may be needed.

Reasoning

"In order to assess reasoning, children will need to communicate their thinking, and this will include using mathematical images, pictures and symbols, as well as explanation. Mathematical talk is indicated by the wording of this aim in the National Curriculum, but this talk is more than using mathematical vocabulary. It requires structuring mathematically and grammatically accurate sentences that communicate the connections that have been made, and convince others that the reasoning is sound.

*Assessment of reasoning must therefore include assessing **how** children have thought about the mathematics, using what they know to make connections and find solutions, rather than just assessing **if** they have been successful in getting correct answers. This will often necessitate listening to the children, using probing questions to get underneath their thinking and expecting them to demonstrate their understanding using different representations; evidence of reasoning is often not contained within maths books, but comes from interaction with the children."*

(NCETM3, Primary Magazine: Issue 82; emphasis [bold] from the original text)

Problem Solving

"In order for problem solving to be assessed, children will need to communicate their process as well as their solution and this will include using mathematical images, pictures and symbols alongside explanation. Pupils may need support to develop the use of different representations when solving problems. In particular, the use of drawings, to both support thinking and communicate thinking, needs to be modelled and should be a focus of some lessons.

Assessment will include looking for efficient and elegant solutions and the application of reasoning and will often necessitate talking to the children, using probing questions to get underneath their thinking. This is vital as, sometimes, children can get correct solutions to problems using inefficient methods or by using incorrect reasoning.

The focus when assessing problem-solving needs to be as much on the method as on the outcome. This starts with how the children make use of what they know from how the problem is presented. Problem-solving should also provoke children to consider 'What if?' leading to the generation of further problems."

(NCETM4, Primary Magazine: Issue 83)

Reception	Example Learning outcomes	Example activities
Working systematically & finding all possibilities	<p>Talk about things being in order.</p> <p>Identify same and different.</p> <p>Use ordinal vocabulary, 1st 2nd etc</p> <p>Sort objects using and explaining criteria</p> <p>Explain what they are thinking and doing.</p> <p>Represent work with objects or pictures and discuss it, talk about ways to check that there are no omissions or repetitions</p>	<p>Sorting activities</p> <ul style="list-style-type: none"> • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose. Make pictures of Billy with his noses. How many different pictures? He has a spotted bowtie and a striped bow tie. Make pictures of Billy with his bow ties. How many different pictures? Can you make different outfits for Billy? Use a nose and a bow tie. How many different outfits? • How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream? Now try with a plain or chocolate cone. A lady bird has six spots. She can have some spots on the left and some on the right. Draw as many different ways of arranging the spots as you can. • Put ten things into 2 paper bags. Can you do it in a different way? • PNS Finding all possibilities: In the café, working in sand, railway track
Generalising & conjecturing Explaining & justifying	<p>Talk about, recognise and recreate simple patterns.</p> <p>Identify same and different.</p> <p>Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices</p> <p>Sort objects using criteria and explaining</p> <p>Make a prediction about the next part of the pattern.</p>	<ul style="list-style-type: none"> • How many smarties in a pack? How many of each colour? Will it be the same for the next pack? Which colour is there more of? • Find different shaped sponges. Which one holds the most water? • Stand up 10 skittles. Have one go at knocking some down with a soft ball or bean bag. Record how many are still standing and how many you knock down. Can you guess how many knocked down before you count them? • Copying, making and talking about patterns with toys, bricks, beads etc • PNS Finding rules and describing patterns: Teddy's presents
Interpreting information & solving logic problems	<p>Recognise similarities and differences.</p> <p>Sort objects using several criteria and sort to their own criteria, justifying their choices.</p> <p>Say why an item does not belong into a set.</p> <p>Guess the criteria being used to sort objects.</p> <p>Explain what they are thinking and doing.</p>	<ul style="list-style-type: none"> • How is your shoe different to your neighbour's? • PNS Logic problems: Shoes, nature sort • Solving everyday problems about classroom tasks e.g. do we have enough apples for snack time?
Reasoning, convincing & proof	<p>Explain why an answer is correct</p> <ul style="list-style-type: none"> • when answering simple problems involving addition and subtraction in their play • why they have used particular shapes in junk modelling • why certain shapes fit into a jigsaw • explain how they work out doubles and halves using resources 	

Year 1	Example Learning outcomes	Example activities
<p>Working systematically & finding all possibilities</p>	<p>Identify same and different. Record different answers in a systematic way, identifying why this is important and explaining how they have done this Explain how answers differ. Recognise that there is sometimes more than one possible answer to a problem. Give examples that match a given statement and those that don't. Talk about patterns in their lists / results.</p>	<ul style="list-style-type: none"> • How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream with a plain or chocolate cone? • Holly and Ivy are two of Santa's elves. Holly wears a red hat and a red tunic. Ivy wears a green hat and a green tunic. In the morning they get dressed in the dark. How many ways can Holly get dressed? • Make a tower of 6 cubes (or a snake or a train) using 2 colours. How many can you make? • Put ten things into 2 paper bags. How many different ways can you do it? If In Teddy Town, teddies are either red or yellow and they live in red or yellow houses. There are 4 teddies - 2 red and 2 yellow, and 4 houses - 2 red and 2 yellow. Can you match each teddy to a house so that the four pairs are all different from each other? • You buy a lollypop for 6p and give the exact money, how many different ways can you pay? • List numbers which total 10 • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? • PNS Finding all possibilities: Lollipops, down the path • Nrich http://nrich.maths.org/9798
<p>Generalising & conjecturing Explaining & justifying</p>	<p>Describe and recreate simple patterns involving numbers, shapes or items. Decide whether examples satisfy given conditions. Describe ways of solving puzzles and problems, explaining choices and decisions. Represent findings orally, using pictures or practically. Make a prediction about the next part of the pattern and explain why. Recognise a simple relationship Make predictions and conjectures</p>	<ul style="list-style-type: none"> • Whose pencil case holds the most? • Whose school bag holds the most? • How many ways can you make a ten using Cuisenaire rods? • PNS Finding rules and describing patterns: Teddy' presents • Nrich http://nrich.maths.org/9009 http://nrich.maths.org/9014 http://nrich.maths.org/8972
<p>Interpreting information & solving logic problems</p>	<p>Use one piece of information and see what effect it has. Check that the answer meets all of the criteria. Solve a problem using given facts.</p>	<ul style="list-style-type: none"> • Shape or number Sudoku 2x2, 3x3 grids • Give me an example of ... and another... eg give me an example of an even number, and another..., a pair of numbers with a sum of ten , and another...etc

	<p>Sort objects, number or shapes and explain why an example does or does not fit into a group</p>	<ul style="list-style-type: none"> • PNS Logic problems: Toys, Granny's garden • Nrich <p>http://nrich.maths.org/9036</p>
<p>Reasoning, convincing & proof</p>	<p>Explain why an answer is correct:</p> <ul style="list-style-type: none"> • showing how they know the multiples of two, five or ten using resources such as numicon or a number line or square • why an number sentence is correct or incorrect using known facts or resources, • why adding or subtracting zero has no effect, • how they know what half or quarter of a quantity object or shape is 	<ul style="list-style-type: none"> • Convince a friend or enemy whether general statements are true or false, for example: <p>All triangles have 3 sides</p> <p>When you add two numbers, you can change the order of the numbers and the answer will be the same</p> <p>You can make 4 different two digit numbers with the digits 2 and 3</p> <p>When you add 10 to a number the units digit stays the same.</p> <p>$3 + 4 = 4 + 3$ (Commutative law)</p> <p>Odd one out: for example with 2D and 3D shape</p> <ul style="list-style-type: none"> • Show me that ... is the same as.... Eg show me that $3 + 4 = 4 + 3$ • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. <p>• http://nrich.maths.org/9016</p>

Year 2	Example Learning outcomes	Example activities
<p><i>Working systematically & finding all possibilities</i></p>	<p><i>Use a systematic way to solve a problem. Create a systematic list of possibilities. Talk about why it is a complete list and how they have been systematic. Look for patterns and possible general statements or relationships</i></p>	<ul style="list-style-type: none"> • <i>If three bears, a red bear, a yellow bear and a green bear, play each other at table tennis, each taking it in turns to play another bear, how many games will there be?</i> • <i>How many different football strips could you make choosing from 2 T shirts and 2 pairs of shorts?</i> • <i>How many different numbers can you make with the digits 1, 2 and 3?</i> • <i>Arrange 3 different coloured smarties in different ways</i> • <i>List pairs of number which have a units digit of 3 when added together</i> • <i>List pairs of numbers with a difference of 3</i> • <i>Use 7 cubes - 5 of them of one colour and 2 of another colour. These 7 have all to be joined together. The five that are of one colour must all touch the table that you are working on. The two that are of a different colour must NOT touch the table. How many different shapes can you find?</i> • <i>PNS Finding all possibilities: Maisie and the maze, line of symmetry</i> • <i>Nrich http://nrich.maths.org/9798</i>
<p><i>Generalising & conjecturing Explaining & justifying</i></p>	<p><i>Identify patterns and relationships involving numbers or shapes, and use these to solve problems. Talk about how a pattern will continue and make predictions. Talk about the pattern generally, discussing a general relationship or statement in words Describe and explain methods, choices and solutions to puzzles and problems.</i></p>	<ul style="list-style-type: none"> • <i>Make a family of multi-link animals, eg a baby dog: How many cubes? Make the next one in the dog family: How many cubes? Make the next members of the dog family How many cubes for each one? How many cubes for the 100th member? Can you see a patterns? How can you work out how many cubes for any dog in the family?</i> • <i>If you fill your pencil case with pennies how rich are you? What about 2pence pieces? 10 pence pieces?</i> • <i>How high is your chair? Your table? Your door? How high would they need to be for a giant child double your height?</i> • <i>If a bank only has 2p and 5p coins, what amounts can you make?</i> • <i>Make multi-link towers of the same size and put them on the corners of a square. How many cubes did you use? Make your towers a different size but keep them all the same. How many now?</i> • <i>Try with a triangle or a pentagon.</i> • <i>PNS Finding rules and describing patterns: Hop scotch grid</i> • <i>http://nrich.maths.org/9009 http://nrich.maths.org/9014 http://nrich.maths.org/8972</i>

<p><i>Interpreting information & solving logic problems</i></p>	<p><i>Solve a problem by identifying given facts and prioritising them. Identify necessary information for solving problems Confirm that they have found the correct solution by checking in another way. Use recording to help them make sense of the information given and to find missing information</i></p>	<ul style="list-style-type: none"> • <i>Give me an example of ... and another... eg give me an example of a pair of numbers with a difference of 2, and another..., a multiple of 3, and another...etc</i> • <i>Shape or number Sudoku 3x3, 4x4 grids</i> • <i>PNS Logic problems: Shape puzzler, sandwich shop</i> • <i>Nrich http://nrich.maths.org/9036</i>
<p><i>Reasoning, convincing & proof</i></p>	<p><i>Explain why an answer is correct,</i></p> <ul style="list-style-type: none"> • <i>use known facts or inverse operations or place value or resources such as dienes or numicon or a number line to show why a number sentence is correct or incorrect,</i> • <i>use resources to show how they know how to find a fraction of a quantity or shape or object and that $2/4 = \frac{1}{2}$</i> • <i>how they have compared and ordered items by measuring</i> • <i>why different combinations of coins might have the same value</i> • <i>why times expressed in different ways may be the same</i> • <i>how they solved problems using pictograms, tallies or block diagrams</i> 	<ul style="list-style-type: none"> • <i>Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true.</i> • <i>Convince a friend or enemy whether these statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples. For example:</i> <i>When you subtract ten from a number, the units digit stays the same</i> <i>You can add 9 to a number by adding 10 and subtracting 1</i> <i>All even numbers end in 0, 2, 4, 6, 8</i> <i>A cube has 9 faces</i> <i>If you have 3 digits, and use each one exactly once in a three digit number, you can make 9 different three digit numbers</i> <i>Etc</i> • <i>Odd one out activities eg looking at three numbers such as 2, 15, 30, decide which is the odd one out and convince your friend</i> • <i>Same and different activities eg 2D and 3D shapes</i> • <i>Show me that ... is the same as... Eg show me that 2 lots of 5 is the same as 5 lots of 2</i> • <i>Nrich http://nrich.maths.org/9016</i>

Year 3	Example Learning outcomes	Example activities
<p>Working systematically & finding all possibilities</p>	<p>Prove that they have found all possible answers by being systematic.</p> <p>Use patterns to make predictions about the number of combinations</p> <p>Use patterns to talk about general statements or relationships</p>	<ul style="list-style-type: none"> • Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? How many outfits if he buys a new nose and bow tie? • List trios of numbers which total 101 • List numbers which leave a remainder when divided by 5 • Find the shapes which straight sides which can be found by cutting a square in to two pieces • PNS Finding all possibilities: fireworks, Susie the snake • Nrich http://nrich.maths.org/9803
<p>Generalising & conjecturing Explaining & justifying</p>	<p>Generate patterns by considering examples systematically in an investigation</p> <p>Make predictions based on patterns in results in an investigation</p> <p>Make general statements and discuss relationships using everyday language</p> <p>Describe and explain methods, choices and solutions to puzzles and problems.</p> <p>Continue more complex patterns.</p>	<ul style="list-style-type: none"> • Draw a 2x2 square on a 100 square. Add the diagonals. What do you notice? Will it always be true? Try different shaped squares/rectangles. • Make a net for a cube. How many different cube nets can you find? • Which numbers can you make using only four 3s and any combinations of operations? • PNS Finding rules and describing patterns: Hop scotch grid, Party bags, L shaped models http://nrich.maths.org/8915 http://nrich.maths.org/8917 http://nrich.maths.org/8909
<p>Interpreting information & solving logic problems</p>	<p>Solve a puzzle by identifying the facts and prioritising them.</p> <p>Use one piece of information in the problem and see what effect it has.</p> <p>Identify necessary information for solving problems</p> <p>Check that their solution meets all the criteria.</p>	<ul style="list-style-type: none"> • Give me an example of ... and another... eg give me an example of a fraction equal to $\frac{1}{2}$, and another..., a pair of numbers which total 100, and another...etc • Shape or number Sudoku, 3x3 grids and sets of 3x3 grids eg 9 x9 • PNS Logic problems: coloured shapes, Rebecca's school day • Nrich http://nrich.maths.org/8944
<p>Reasoning, convincing & proof</p>	<p>Explain why an answer is correct,</p> <ul style="list-style-type: none"> • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect, • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, demonstrating that ten units is one ten and ten tens is one hundred 	<ul style="list-style-type: none"> • Convince a friend or enemy whether these statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples. For example: Any odd number is one more than an even number Any even number can be made as the sum of two odd numbers The multiples of 4 are always even etc • Odd one out activities

- Use resources to show how they know what one tenth of a number is
- Use resources or pictures to show how they know what a fraction of a number is and to show equivalent fractions
- How they know what the perimeter of a shape is
- Why times expressed in different ways may be the same
- How they use conversions between metric units of measurements to solve problems (eg m,, cm, mm, kg, g, l ml)
- Why a full turn is the same as four quarter turns etc
- How they solved problems using bar charts, pictograms and tables

- Same and Different Activities
- Show me that ... is the same as.... Eg show me that a litre is the same as two lots of 500 ml
- Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true.
- Nrich
<http://nrich.maths.org/8921>
(Link to persuasive language)

Year 4	Example Learning outcomes	Example activities
<p><i>Working systematically & finding all possibilities</i></p>	<p><i>Solve a problem by checking possible solutions against a given criteria.</i></p> <p><i>List possible answers in a systematic way efficiently.</i></p> <p><i>Justify the approach as being systematic.</i></p> <p><i>Prove that all items are listed</i></p> <p><i>Make a general statement and provide a convincing argument that it is true.</i></p> <p><i>Use a pattern to predict the next number of combinations</i></p>	<ul style="list-style-type: none"> • <i>How many different ice creams can you make if you choose one scoop of either chocolate or strawberry ice cream with a plain or chocolate cone? Add in other flavours of ice cream, different types of cone, and then chocolate or toffee sauce on top.</i> • <i>List numbers which leave a remainder of 1 when divided by 7</i> • <i>PNS Finding all possibilities: sheep dog trails, 3 digits</i> • <i>Nrich</i> <p>http://nrich.maths.org/9803</p>
<p><i>Generalising & conjecturing</i></p> <p><i>Explaining & justifying</i></p>	<p><i>Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols</i></p> <p><i>Use patterns to make predictions and general statements.</i></p> <p><i>Talk about the justification for the general statement.</i></p> <p><i>Describe and continue more complex patterns.</i></p> <p><i>Draw conclusions from investigations and explain their reasoning</i></p>	<ul style="list-style-type: none"> • <i>How many squares on a chess board?</i> • <i>Add three consecutive numbers. What do you notice about the answer? Now try adding 5, 7, 9 consecutive numbers.</i> • <i>Find the number of vertices, faces and edges on some 3D shapes. Do you notice a pattern? Is there a separate pattern for prisms and pyramids?</i> • <i>PNS Finding rules and describing patterns: Party bags, L shaped models</i> • <i>Nrich</i> <p>http://nrich.maths.org/8915</p> <p>http://nrich.maths.org/8917</p> <p>http://nrich.maths.org/8909</p>
<p><i>Interpreting information & solving logic problems</i></p>	<p><i>Solve a problem by identifying and prioritising given facts and information, checking possible solutions against given criteria.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Solve a problem by identifying and prioritising given facts and information.</i></p>	<ul style="list-style-type: none"> • <i>Give me an example of ... and another... eg give me an example of a rectangle with perimeter of 24cm, and another...., three consecutive numbers with an odd total, and another...etc</i> • <i>Think of a number.....</i> <p><i>Double it, add 15 subtract 3, halve it, take away the number you first thought of. Now I will read your mind, the answer is 6!</i></p> <p><i>Why does it work?</i></p> <p><i>Make up your own</i></p> <ul style="list-style-type: none"> • <i>PNS Logic problems: shape puzzle, boys and girls</i> • <i>Nrich</i> <p>http://nrich.maths.org/8944</p>
<p><i>Reasoning, convincing & proof</i></p>	<p><i>Explain why an answer is correct,</i></p> <ul style="list-style-type: none"> • <i>use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect</i> • <i>Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction,</i> 	<ul style="list-style-type: none"> • <i>Convince a friend or enemy whether general statements are true or false. Explain their thinking, showing why a general statement may be true or not true with the use of particular examples and mathematical patterns and properties. For example:</i> <p><i>Any odd number is double a number add 1</i></p> <p><i>If you multiply a number by 10 the digits move one place to the left</i></p>

- Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers
- Explain what they know about multiplying by 0 and 1, and dividing by 1
- Use an array to explain how to find factors of a number, and how to multiply two or three digit number by a one digit number using the distributive law
- Use resources or diagrams to show equivalent fractions and how to find a non unit fraction of a quantity or shape
- how they use conversions between metric units of measurements to solve problems (eg km, m, hour, minute)
- how they found the area of a shape
- why analogue and digital, and 12 and 24 hour times might be the same

The number of lines of reflective symmetry in a regular polygon is equal to the number of sides of the polygon

The sum of three odd numbers is odd

- Odd one out activities
- Same and different activities for example with 2D and 3D shapes
- Show me that ... is the same as.... Eg show me that $\frac{1}{4}$ of 24 is 6
- Explain why odd numbers added to odd numbers have even totals etc
- Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true.

• Nrich

<http://nrich.maths.org/8921>

Year 5	Example Learning outcomes	Example activities
<p><i>Working systematically & finding all possibilities</i></p>	<p><i>Find all possibilities by working systematically. Prove all possibilities are listed Recognise when reasoning is systematic and when it is not. Identify a pattern to make a prediction of the number of possibilities. Make a general statement and provide a convincing argument and apply this to other situations</i></p>	<ul style="list-style-type: none"> • <i>Billy the clown wears a coloured nose and bowtie for his show. He has a red nose and a blue nose, and a spotted bowtie and a striped bow tie. How many different outfits can he appear in? How many outfits if he buys a new nose and bow tie? What about if he decides to wear a hat as well and buys a yellow and an orange hat too? Shoes too? List the factors of for example 48, how do you know you have them all?</i> • <i>List the square numbers between 50 and 500</i> • <i>Place each of the numbers 1 to 5 in a V shape so that the two arms of the V have the same total. How many different possibilities are there? What do you notice about all the solutions you find? Can you explain what you see? Can you convince someone that you have all the solutions? What happens if we use the numbers from 2 to 6? From 12 to 16? From 37 to 41? From 103 to 107? What can you discover about a V that has arms of length 4 using the numbers 1-7?</i> • <i>PNS Finding all possibilities: ice creams, treasure hunt</i> • <i>Nrich</i> http://nrich.maths.org/9803
<p><i>Generalising & conjecturing Explaining & justifying</i></p>	<p><i>Generate patterns through systematic examples in an investigation identify and describe patterns using mathematical language Accurately predict a later term in a pattern or sequence Use a pattern to suggest and test general statements. Provide a convincing argument for the general statement. Draw conclusions from investigations and explain their reasoning using words, symbols or diagrams as appropriate</i></p>	<ul style="list-style-type: none"> • <i>The Tower of Hanoi Move all the discs to the right hand tower. Only move one disc at a time. Never put a large disc on a smaller one. What is the smallest number of moves? Try different numbers of different sized disks.</i> • <i>If you have 3 towns, and each one has one road to the others, how many roads? How many roads for 4, 5, 6, any number of towns?</i> • <i>Explore the digit roots of numbers. To find the digit root, add the digits together. If your total has more than one digit root, continue to add the digits together. When your total has one digit, this is the digit root. What do you notice? What are the digit roots of the multiples of 3?</i> • <i>PNS Finding rules and describing patterns: candle problem, sequence of models</i> • <i>Nrich</i> http://nrich.maths.org/8915 http://nrich.maths.org/8917

<p><i>Interpreting information & solving logic problems</i></p>	<p><i>Use one piece of information in more complex problems and see what effect it has.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Check that the answer meets the criteria.</i></p> <p><i>Choose and use a recording system to organise the given information independently.</i></p> <p><i>Use appropriate language that is associated with this type of logic problem, e.g. 'If this ... then this will change ...'</i></p>	<ul style="list-style-type: none"> • <i>Give me an example of ... and another... eg give me an example of two fractions with a total of 2, and another..., a 3D shape with at least two triangular faces , and another...etc</i> <i>Andrea, Peter, Debra and Simon are each wearing one of black, red, yellow and green T-shirts. Use the following clues to find out which colour shirt each person is wearing</i> <i>The red shirt is worn by one of the boys</i> <i>Andrea and the girl who always wears black are in different schools</i> <i>Simon's shirt colour has the same number of letters as his name</i> • <i>PNS Logic problems: nicknames, tea for two</i> • <i>Nrich http://nrich.maths.org/8944</i>
<p><i>Reasoning, convincing & proof</i></p>	<p><i>Explain why an answer is correct, for</i></p> <ul style="list-style-type: none"> • <i>use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect</i> • <i>Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction,</i> • <i>Use an array to show the distributive law and use this to explain their written methods for long multiplication</i> • <i>Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers</i> • <i>Explain common factors and multiples using an array, number line or resources</i> • <i>Prove whether a number is prime or not using an array or resources or known facts</i> • <i>Use resources or diagrams to show equivalent fractions and how to add and subtract fractions with denominators which are the same or multiples of the same number</i> • <i>how they use conversions between metric units and between metric and imperial units of measurements to solve problems</i> • <i>how they use facts about angles at a point or making a straight line to solve problems</i> • <i>how they solve problems using line graphs and tables</i> 	<ul style="list-style-type: none"> • <i>Convince a friend or enemy whether these general are true or false or sometimes true. Explain their thinking, with the use of particular examples and mathematical patterns and properties. For example:</i> <i>A multiple of 6 is a multiple of 2 and 3</i> <i>The digits of multiples of nine add up to 9</i> <i>The product of two consecutive numbers is even</i> <i>Angles on a straight line add up to 180 degrees</i> • <i>Odd one out activities eg 2D and 3D shape</i> • <i>Same and different activities eg 2D and 3D shape</i> • <i>Show me that ... is the same as.... Eg show me that 1/5 of 10 is the same as $\frac{1}{2}$ of 4</i> • <i>Show me why adding consecutive odd number from 1 makes square numbers eg $1 + 3 + 5 = 9$ (picture proof)</i> • <i>Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true.</i> • <i>Nrich http://nrich.maths.org/8921</i>

Year 6	Example Learning outcomes	Example activities
<p><i>Working systematically & finding all possibilities</i></p>	<p><i>Identify a pattern to make a prediction of the number of possibilities.</i></p> <p><i>Make a general statement with a convincing argument and apply this to other situations with similar or more combinations.</i></p> <p><i>Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.</i></p>	<ul style="list-style-type: none"> • <i>How many ways can three children line up for assembly?</i> <i>Four children?</i> <i>Ten children?</i> • <i>List fractions with the same value as 0.01</i> • <i>List sets of three numbers with a mean of 6</i> • <i>List primes between 50 and 70</i> • <i>If the final score at the end of a hockey match was 4,2, what could the score be at half time?</i> • <i>PNS Finding all possibilities: King Arnold, 4 by 4</i> • <i>Nrich http://nrich.maths.org/9803</i>
<p><i>Generalising & conjecturing</i></p> <p><i>Explaining & justifying</i></p>	<p><i>Construct and use a general statement in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence).</i></p> <p><i>Draw conclusions from investigations and explain their reasoning</i></p> <p><i>Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.</i></p>	<ul style="list-style-type: none"> • <i>How many handshakes take place if 30 people in a room shake hands with each other exactly once?</i> • <i>Make a $3 \times 3 \times 3$ cube out of 27 small cubes. Imagine dipping it into paint. How many small cubes have: 3 faces painted? 2 faces painted? 1 face painted? 0 faces painted?</i> <i>Investigate for $1 \times 1 \times 1$, $2 \times 2 \times 2$ and other sized cubes</i> • <i>Investigating regions:</i> <i>Draw a circle and put two dots anywhere on the circumference. Join these up with straight lines and count how many regions you make. Try other number of dots.</i> • <i>Which numbers have odd totals of factors?</i> • <i>PNS Finding rules and describing patterns: candle problem, sequence of models</i> • <i>Nrich http://nrich.maths.org/8915</i> <i>http://nrich.maths.org/8917</i> <i>http://nrich.maths.org/8909</i>
<p><i>Interpreting information & solving logic problems</i></p>	<p><i>Identify necessary information for solving problems</i></p> <p><i>Prioritise and use given facts to solve and check complex logic problems.</i></p> <p><i>Ask 'What if . . . ?' questions.</i></p> <p><i>Recognise the effect of extensions such as 'What if ...?' questions.</i></p> <p><i>Create their own criteria for solving a logic problem in the context of a solved problem</i></p> <p><i>Refine and extend problems to generate fuller solutions</i></p>	<ul style="list-style-type: none"> • <i>Give me an example of ... and another... eg give me an example of a fractions equivalent to $\frac{3}{4}$, and another...., a fraction smaller than $\frac{1}{10}$, and another...etc</i> • <i>Crossing the bridge</i> <i>Four friends need to cross a bridge.</i> <i>They start on the same side of the bridge. A maximum of two people can cross at any time. It is night and they have just one lamp. People that cross the bridge must carry the lamp to see the way. A pair must walk together at the rate of the slower person:</i>

		<p>Rachel: - takes 1 minute to cross Ben: - takes 2 minutes to cross George: - takes 7 minutes to cross Yvonne: - takes 10 minutes to cross</p> <p>The second fastest solution gets the friends across in 21 minutes. The fastest takes 17 minutes. Can you work out how it is done?</p> <ul style="list-style-type: none"> • PNS Logic problems: Albert square, house points • Nrich <p>http://nrich.maths.org/8944</p> <ul style="list-style-type: none"> • Murder mystery <p>https://www.ncetm.org.uk/resources/20330</p>
<p>Reasoning, convincing & proof</p>	<p>Explain why an answer is correct, using concise argument, involving symbols, mathematical language, graphs or diagrams.</p> <ul style="list-style-type: none"> • use known facts or inverse operations or place value to show why a number sentence is correct or incorrect • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, • Use an array to show the distributive law and use this to explain long multiplication • Explain how they perform long and short division, using resources such as place value counters • Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers • Use resources or diagrams to show equivalent fractions and how to order, add, subtract and multiply fractions with different denominators and divide fractions by whole numbers • Explain how they solve ratio and proportion problems, perhaps using the bar method • Explain when they can use the formulae for area and volume of shapes • How to generate number sequences, and the rule for sequences they have generated • How they express missing number problems algebraically 	<ul style="list-style-type: none"> • Convince a friend or an enemy that general statements are always, sometimes or never true. If never true, disprove by counter example. Use particular examples but recognise that arguments should be based on general mathematical patterns and properties. For example: If you add three consecutive numbers the sum is three times the middle number Multiplying does not always make the answer larger Dividing a whole number by half makes the answer twice as big Rectangles always have two diagonals which meet at right angles • Odd one out activities eg 2D and 3D shape • Same and different activities: eg 2D and 3D shapes • Show me that ... is the same as.... Eg show me that 30% of 60 is the same as 60% of 30 • Explain why odd numbers multiplied by even numbers are odd etc • Explain why opposite angles are equivalent • Explain why the general patterns or rules they found as part of 'finding rules and describing patterns' are true. <p>Nrich</p> <p>http://nrich.maths.org/89214</p>

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| | <ul style="list-style-type: none">• <i>How they use conversions between metric units (miles and km) and between metric and imperial units of measurements to solve problems</i>• <i>How they use facts about angles in a shape, at a point or vertically opposite to solve problems</i>• <i>How they solve problems using pie charts and line graphs, and calculate and interpret mean</i> | |
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Working systematically Progression

YR	<p><i>Talk about things being in order.</i></p> <p><i>Identify same and different.</i></p> <p><i>Use ordinal vocabulary, 1st 2nd etc</i></p> <p><i>Sort objects using and explaining criteria</i></p> <p><i>Explain what they are thinking and doing.</i></p> <p><i>Represent work with objects or pictures and discuss it, talk about ways to check that there are no omissions or repetitions</i></p>
Y1	<p><i>Identify same and different.</i></p> <p><i>Record different answers in a systematic way, identifying why this is important and explaining how they have done this</i></p> <p><i>Explain how answers differ.</i></p> <p><i>Recognise that there is sometimes more than one possible answer to a problem.</i></p> <p><i>Give examples that match a given statement and those that don't.</i></p> <p><i>Talk about patterns in their lists / results.</i></p>
Y2	<p><i>Use a systematic way to solve a problem.</i></p> <p><i>Create a systematic list of possibilities.</i></p> <p><i>Talk about why it is a complete list and how they have been systematic.</i></p> <p><i>Look for patterns and possible general statements or relationships</i></p>
Y3	<p><i>Prove that they have found all possible answers by being systematic.</i></p> <p><i>Use patterns to make predictions about the number of combinations</i></p> <p><i>Use patterns to talk about general statements or relationships</i></p>
Y4	<p><i>Solve a problem by checking possible solutions against a given criteria.</i></p> <p><i>List possible answers in a systematic way efficiently.</i></p> <p><i>Justify the approach as being systematic.</i></p> <p><i>Prove that all items are listed</i></p> <p><i>Make a general statement and provide a convincing argument that it is true.</i></p> <p><i>Use a pattern to predict the next number of combinations</i></p>
Y5	<p><i>Find all possibilities by working systematically.</i></p> <p><i>Prove all possibilities are listed</i></p> <p><i>Recognise when reasoning is systematic and when it is not.</i></p> <p><i>Identify a pattern to make a prediction of the number of possibilities.</i></p> <p><i>Make a general statement and provide a convincing argument and apply this to other situations</i></p>
Y6	<p><i>Identify a pattern to make a prediction of the number of possibilities.</i></p> <p><i>Make a general statement with a convincing argument and apply this to other situations with similar or more combinations.</i></p> <p><i>Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.</i></p>

Generalising & conjecturing, Explaining & justifying Progression

YR	<p>Talk about, recognise and recreate simple patterns. Identify same and different. Describe solutions to practical problems, drawing on experience, talking about their own ideas, methods and choices Sort objects using criteria and explaining Make a prediction about the next part of the pattern.</p>
Y1	<p>Describe and recreate simple patterns involving numbers, shapes or items. Decide whether examples satisfy given conditions. Describe ways of solving puzzles and problems, explaining choices and decisions. Represent findings orally, using pictures or practically. Make a prediction about the next part of the pattern and explain why. Recognise a simple relationship Make predictions and conjectures</p>
Y2	<p>Identify patterns and relationships involving numbers or shapes, and use these to solve problems. Talk about how a pattern will continue and make predictions. Talk about the pattern generally, discussing a general relationship or statement in words Describe and explain methods, choices and solutions to puzzles and problems.</p>
Y3	<p>Generate patterns by considering examples systematically in an investigation Make predictions based on patterns in results in an investigation Make general statements and discuss relationships using everyday language Describe and explain methods, choices and solutions to puzzles and problems. Continue more complex patterns.</p>
Y4	<p>Report solutions to puzzles and problems, giving explanations and reasoning orally and in writing, using diagrams and symbols Use patterns to make predictions and general statements. Talk about the justification for the general statement. Describe and continue more complex patterns. Draw conclusions from investigations and explain their reasoning</p>
Y5	<p>Generate patterns through systematic examples in an investigation identify and describe patterns using mathematical language Accurately predict a later term in a pattern or sequence Use a pattern to suggest and test general statements. Provide a convincing argument for the general statement. Draw conclusions from investigations and explain their reasoning using words, symbols or diagrams as appropriate</p>
Y6	<p>Construct and use a general statement in words then symbols (e.g. the cost of c pens at 15 pence each is $15c$ pence). Draw conclusions from investigations and explain their reasoning Express the general statement from an investigation using mathematical language, symbols and sometimes with algebra.</p>

Interpreting information & solving logic problems Progression

YR	<p><i>Recognise similarities and differences.</i></p> <p><i>Sort objects using several criteria and sort to their own criteria, justifying their choices.</i></p> <p><i>Say why an item does not belong into a set.</i></p> <p><i>Guess the criteria being used to sort objects.</i></p> <p><i>Explain what they are thinking and doing.</i></p>
Y1	<p><i>Use one piece of information and see what effect it has.</i></p> <p><i>Check that the answer meets all of the criteria.</i></p> <p><i>Solve a problem using given facts.</i></p> <p><i>Sort objects, number or shapes and explain why an example does or does not fit into a group</i></p>
Y2	<p><i>Solve a problem by identifying given facts and prioritising them.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Confirm that they have found the correct solution by checking in another way.</i></p> <p><i>Use recording to help them make sense of the information given and to find missing information</i></p>
Y3	<p><i>Solve a puzzle by identifying the facts and prioritising them.</i></p> <p><i>Use one piece of information in the problem and see what effect it has.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Check that their solution meets all the criteria.</i></p>
Y4	<p><i>Solve a problem by identifying and prioritising given facts and information, checking possible solutions against given criteria.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Solve a problem by identifying and prioritising given facts and information.</i></p>
Y5	<p><i>Use one piece of information in more complex problems and see what effect it has.</i></p> <p><i>Identify necessary information for solving problems</i></p> <p><i>Check that the answer meets the criteria.</i></p> <p><i>Choose and use a recording system to organise the given information independently.</i></p> <p><i>Use appropriate language that is associated with this type of logic problem, e.g. 'If this ... then this will change ...'</i></p>
Y6	<p><i>Identify necessary information for solving problems</i></p> <p><i>Prioritise and use given facts to solve and check complex logic problems.</i></p> <p><i>Ask 'What if . . . ?' questions.</i></p> <p><i>Recognise the effect of extensions such as 'What if ...?' questions.</i></p> <p><i>Create their own criteria for solving a logic problem in the context of a solved problem</i></p> <p><i>Refine and extend problems to generate fuller solutions</i></p>

Reasoning, convincing & proof Progression

YR	<p>Explain why an answer is correct</p> <ul style="list-style-type: none"> • when answering simple problems involving addition and subtraction in their play • why they have used particular shapes in junk modelling • why certain shapes fit into a jigsaw • explain how they work out doubles and halves using resources
Y1	<p>Explain why an answer is correct:</p> <ul style="list-style-type: none"> • showing how they know the multiples of two, five or ten using resources such as numicon or a number line or square • why a number sentence is correct or incorrect using known facts or resources, • why adding or subtracting zero has no effect, • how they know what half or quarter of a quantity object or shape is
Y2	<p>Explain why an answer is correct,</p> <ul style="list-style-type: none"> • use known facts or inverse operations or place value or resources such as dienes or numicon or a number line to show why a number sentence is correct or incorrect, • use resources to show how they know how to find a fraction of a quantity or shape or object and that $\frac{2}{4} = \frac{1}{2}$ • how they have compared and ordered items by measuring • why different combinations of coins might have the same value • why times expressed in different ways may be the same • how they solved problems using pictograms, tallies or block diagrams
Y3	<p>Explain why an answer is correct,</p> <ul style="list-style-type: none"> • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect, • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, demonstrating that ten units is one ten and ten tens is one hundred • Use resources to show how they know what one tenth of a number is • Use resources or pictures to show how they know what a fraction of a number is and to show equivalent fractions • How they know what the perimeter of a shape is • Why times expressed in different ways may be the same • How they use conversions between metric units of measurements to solve problems (eg m., cm, mm, kg, g, l ml) • Why a full turn is the same as four quarter turns etc • How they solved problems using bar charts, pictograms and tables
Y4	<p>Explain why an answer is correct,</p> <ul style="list-style-type: none"> • use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect • Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction, • Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers • Explain what they know about multiplying by 0 and 1, and dividing by 1 • Use an array to explain how to find factors of a number, and how to multiply two or three digit number by a one digit number using the distributive law • Use resources or diagrams to show equivalent fractions and how to find a non unit fraction of a quantity or shape • how they use conversions between metric units of measurements to solve problems (eg km, m, hour, minute) • how they found the area of a shape • why analogue and digital, and 12 and 24 hour times might be the same

Y5	<p><i>Explain why an answer is correct, for</i></p> <ul style="list-style-type: none"> • <i>use known facts or inverse operations or place value or resources such as dienes or a number line to show why a number sentence is correct or incorrect</i> • <i>Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction,</i> • <i>Use an array to show the distributive law and use this to explain their written methods for long multiplication</i> • <i>Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers</i> • <i>Explain common factors and multiples using an array, number line or resources</i> • <i>Prove whether a number is prime or not using an array or resources or known facts</i> • <i>Use resources or diagrams to show equivalent fractions and how to add and subtract fractions with denominators which are the same or multiples of the same number</i> • <i>how they use conversions between metric units and between metric and imperial units of measurements to solve problems</i> • <i>how they use facts about angles at a point or making a straight line to solve problems</i> • <i>how they solve problems using line graphs and tables</i>
Y6	<p><i>Explain why an answer is correct, using concise argument, involving symbols, mathematical language, graphs or diagrams.</i></p> <ul style="list-style-type: none"> • <i>use known facts or inverse operations or place value to show why a number sentence is correct or incorrect</i> • <i>Use resources such as dienes and place value counters to show how they used column methods for addition and subtraction,</i> • <i>Use an array to show the distributive law and use this to explain long multiplication</i> • <i>Explain how they perform long and short division, using resources such as place value counters</i> • <i>Explain how they solved word problems: choosing operations and disregarding unnecessary information and checking their answers</i> • <i>Use resources or diagrams to show equivalent fractions and how to order, add, subtract and multiply fractions with different denominators and divide fractions by whole numbers</i> • <i>Explain how they solve ratio and proportion problems, perhaps using the bar method</i> • <i>Explain when they can use the formulae for area and volume of shapes</i> • <i>How to generate number sequences, and the rule for sequences they have generated</i> • <i>How they express missing number problems algebraically</i> • <i>How they use conversions between metric units (miles and km) and between metric and imperial units of measurements to solve problems</i> • <i>How they use facts about angles in a shape, at a point or vertically opposite to solve problems</i> • <i>How they solve problems using pie charts and line graphs, and calculate and interpret mean</i>